The paper was set in accordance to the UNEB setting system covering the sections of VECTORS, TRIGONOMETRY, ANALYSIS, GEOMETRY and ALGEBRA.

1. Solve the equation $2\cos 2\theta + 3\sin\theta = 1;\ 0^o \le x \le 360^o$.

$$2(1-2\sin^2\theta)+3\sin\theta = 1, \quad 4\sin^2\theta - 3\sin\theta - 1 = 0$$
$$4\sin^2\theta - 4\sin\theta + \sin\theta - 1 = 0, \quad (4\sin\theta+1)(\sin\theta-1)=0$$
$$\sin\theta = -\frac{1}{4},\ \theta = 94.48^o,\ 345.52^o, \quad \sin\theta = 1,\ \theta = 90^o$$

Comment: **Popular question as the students did well to use the double angles but they were denied marks for using calculators to solve the quadratic equation.**

2. Differentiate from first principles: $y = 3x^2 + \cos 2x$.

$$y + \partial y = 3(x+\partial x)^2 + \cos 2(x+\partial x)$$
$$\partial y = 3(x+\partial x)^2 + \cos 2(x+\partial x) - (3x^2 + \cos 2x)$$
$$\partial y = 3x^2 + 6x\partial x + 3(\ \partial x)^2 + -2\sin(2x+\partial x)\sin\partial x - 3x^2$$
$$\frac{\partial y}{\partial x} = 6x + 3\partial x - 2\sin(2x+\partial x), \qquad \frac{dy}{dx} = 6x - 2\sin 2x$$

Comment: **The students did not recall the concepts of differentiating from first principles. Poorly done amongst the students.**

3. Express in partial fractions: $\dfrac{3x+2}{(2x-1)^2(3-x)}$

Let $\dfrac{3x+2}{(2x-1)^2(3-x)} \equiv \dfrac{A}{(2x-1)} + \dfrac{B}{(2x-1)^2} + \dfrac{C}{(3-x)}$

$$3x+2 \equiv A(2x-1)(3-x) + B(3-x) + C(2x-1)^2$$

$A = \dfrac{22}{25},\ B = \dfrac{7}{5},\ C = \dfrac{11}{25}$ $\qquad \dfrac{3x+2}{(2x-1)^2(3-x)} \equiv \dfrac{22}{25(2x-1)} + \dfrac{7}{5(2x-1)^2} + \dfrac{11}{25(3-x)}$

Comment: **Partial fractions was a popular question but the concept of LINEAR REPEATED FACTOR was confused with a QUADRATIC FACTOR.**

4. Prove that in any triangle ABC, $\dfrac{a^2-b^2}{c^2} = \dfrac{\sin(A-B)}{\sin(A+B)}$

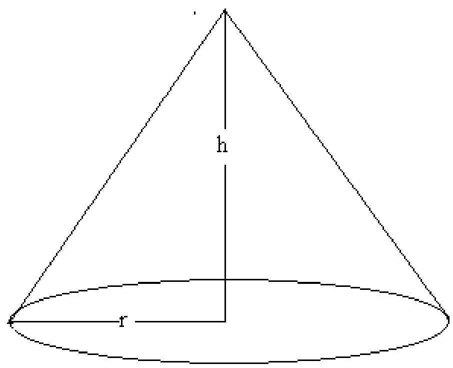
From L.H.S $\dfrac{a^2-b^2}{c^2} = \dfrac{4R^2(\sin^2 A - \sin^2 B)}{4R^2\sin^2 C}$ $\qquad \sin(A+B) = \sin C$

$$= \frac{(\sin A + \sin B)(\sin A - \sin B)}{\sin^2(A+B)} \qquad = \frac{\left(2\sin\frac{A+B}{2}\cos\frac{A-B}{2}\right)\left(2\cos\frac{A+B}{2}\sin\frac{A-B}{2}\right)}{\sin^2(A+B)}$$

$$= \frac{\left(2\sin\frac{A+B}{2}\cos\frac{A+B}{2}\right)\left(2\cos\frac{A+B}{2}\sin\frac{A+B}{2}\right)}{\sin^2(A+B)} \qquad = \frac{\sin(A+B)\sin(A-B)}{\sin^2(A+B)} \quad = \frac{\sin(A-B)}{\sin(A+B)}$$ as R.H.S

Comment: **Poorly attempted. Application of the sine rule. The same question appeared in UNEB Maths paper 1 2011 in section B.**

5. A right circular cone is to be made such that its slant height is π metres. Show that the maximum volume of the cone is $\frac{2\pi^4}{9\sqrt{3}}$.



Solution: $\pi^2 = r^2 + h^2 \qquad \Rightarrow \quad r^2 = \pi^2 - h^2$

Volume $V = \frac{\pi}{3} r^2 h$ so $V = \frac{\pi}{3} h(\pi^2 - h^2)$

$\frac{dV}{dh} = \frac{\pi}{3}(\pi^2 - 3h^2)$ but for max

Volume, $\frac{dV}{dh} = 0$ So, $\frac{\pi}{3}(\pi^2 - 3h^2) = 0 \quad h = \frac{\pi}{\sqrt{3}}$

$\therefore$ Maximum Volume

$V = \frac{\pi}{3}(\pi^2 - \frac{\pi^2}{3})\frac{\pi}{\sqrt{3}}$ $= \frac{2\pi^4}{9\sqrt{3}}$.

Comment: **Application of maximum and minimum was poorly done. Need to revise the topic.**

6. The gradient of a curve at a point (x, y) is $\frac{x^3 - 8}{x^2}$. Find the equation of the curve given that it passes through the point $(2, 4)$.

$\frac{dy}{dx} = \frac{x^3 - 8}{x^2}$, $\frac{dy}{dx} = x - \frac{8}{x^2} \quad y = \int x - 8x^{-2}\, dx$

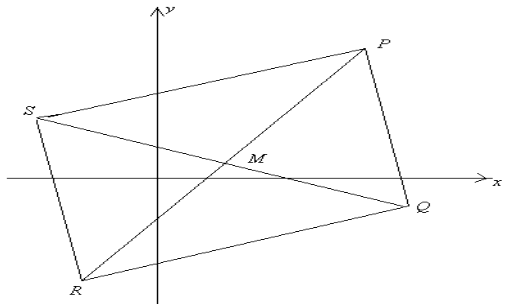
$y = \frac{x^2}{2} + \frac{8}{x} + c$, $x = 2, \ y = 4$, so $4 = 2 + 4 + c$, thusb $c = -2$

Equation of the curve is $y = \frac{x^2}{2} + \frac{8}{x} - 2$

Comment: **Analysis required integration but after splitting the terms. Most of the students failed to interpret the question and begun differentiating, sketching the curve e.t.c indicating that most were living on cram work.**

7. The gradient of the side PQ of the rectangle $PQRS$ is $3/4$. The coordinates of the opposite corners Q, S are respectively $(6, 3)$ and $(-5, 1)$. Find the equation

of PR.



Find the eqn of $\overline{PQ} = \dfrac{y-3}{x-6} = \dfrac{3}{4}$, $\Rightarrow\ 4y = 3x - 6$

Find the eqn of $\overline{RS} = \dfrac{y-1}{x+5} = -\dfrac{4}{3}$, $\Rightarrow\ 3y = -4x - 17$ since they are perpendicular.

Solving the two simultaneously, we get the coordinates of P as $P(-2,\ -3)$

Let M be the midpoint of $\overline{QS}$, thus the coordinates are $M\left(\frac{1}{2},\ 2\right)$.

M is also the midpoint of $\overline{PR}$, gradient of $\overline{PR} = \dfrac{-3-2}{-2-\frac{1}{2}} = 2$

$\therefore$ Equation of $\overline{PR}$: $\dfrac{y+3}{x+2} = 2$, giving $y = 2x + 1$

Comment: **Coordinate geometry was poorly done despite the question appearing so many times. The students seem not to revise the work given to them.**

8. Find the angle between the line $\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{3-z}{-4}$ and the plane $4x + 3y + 2z + 1 = 0$.

$d = 2i + 3j + 4k$, is the direction vector of the line.

$n = 4i + 3j + 2k$, vector normal to the plane.

$$\cos(90^o - \theta) = \frac{\begin{pmatrix}2\\3\\4\end{pmatrix}.\begin{pmatrix}4\\3\\2\end{pmatrix}}{\sqrt{4+9+16}.\sqrt{16+9+4}} \qquad = \frac{8+9+8}{\sqrt{29}.\sqrt{29}} = \frac{25}{29} \qquad (90^o - \theta) = 30.45^o \ \therefore\ \theta = 59.55^o$$

Comment: **The question was popular but the students refused to indicate vector symbols and they also failed to read the direction vector of the line correctly.**

9a) Solve the equation: $\cos 3x - \sin 2x = \sin 3x - \cos 2x$; $0 \le x \le \pi$

$$\cos 3x + \cos 2x = \sin 3x + \sin 2x \qquad 2\cos\frac{5x}{2}\cos\frac{x}{2} - 2\sin\frac{5x}{2}\cos\frac{x}{2} = 0$$

$$2\cos\frac{x}{2}\left(\cos\frac{5x}{2} - \sin\frac{5x}{2}\right) = 0 \qquad 2\cos\frac{x}{2} = 0,\ \frac{x}{2} = 90^o,\ x = \pi$$

$\left(\cos\frac{5x}{2} - \sin\frac{5x}{2}\right) = 0 \qquad \tan\frac{5x}{2} = 1, \qquad \frac{5x}{2} = 45^o, 225^o, 405^o$

$\frac{5x}{2} = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \quad x = \frac{\pi}{10}, \frac{\pi}{2}, \frac{9\pi}{10}$

Comment: **The question had appeared in their MOT but like earlier mentioned they do not revise what is given to them. Those who managed failed to convert the angles to radians.**

b) Express $10\sin x\cos x + 12\cos 2x$ in the form $R\sin(2x+\alpha)$, hence or otherwise solve $10\sin x\cos x + 12\cos 2x + 7 = 0$ in the range $0^o \le x \le 360^o$.

$10\sin x\cos x + 12\cos 2x = 5\sin 2x + 12\cos 2x$

Let $5\sin 2x + 12\cos 2x \equiv R\sin 2x\cos\alpha + R\cos 2x\sin\alpha$

$\Rightarrow 5 = R\cos\alpha,\ 12 = R\sin\alpha$, thus $\tan\alpha = \frac{12}{5} \quad \therefore \alpha = 67.38^o$

$R = \sqrt{5^2 + 12^2} = 13$

$5\sin 2x + 12\cos 2x = 13\sin(2x + 67.38^o)$ as required.

$10\sin x\cos x + 12\cos 2x + 7 = 0$, $13\sin(2x + 67.38^o) = -7$

$2x + 67.38^o = 212.59^o,\ 327.41^o, \quad 2x = 145.21^o,\ 260.03^o$

Thus, $x = 72.61^o,\ 130.02^o$

Comment: **Trigonometry was not popular despite the question being set again.**

10a) Use the substitution $y = x - \frac{2}{x}$ to solve the equation $x^4 + x^3 - 16x^2 - 2x + 4 = 0$.

$x^4 + x^3 - 16x^2 - 2x + 4 = 0$ Divide thru by x^2

$x^2 + x - 16 - \frac{2}{x} + \frac{4}{x^2} = 0 \qquad x^2 + \frac{4}{x^2} + \left(x - \frac{2}{x}\right) - 16 = 0$

$y^2 + 4 = x^2 + \frac{4}{x^2} \qquad y^2 + 4 + y - 16 = 0$

$y^2 + y - 12 = 0, \quad (y-3)(y+4) = 0, \quad y = -4,\ y = 3$

Thus $x^2 + 4x - 2 = 0$, $x = \frac{-4 \pm \sqrt{24}}{2}$ And $x^2 - 3x - 2 = 0$, $x = \frac{3 \pm \sqrt{17}}{2}$

Comment: **The question was popular as these were taught in s5 first term. The students who used calculators were denied the marks. Popular question, a MUST do to all.**

b) Solve the equations: $\frac{x+2y}{-3} = \frac{y+2z}{4} = \frac{2x+z}{5},\ x + y + z = 2$

Let $\frac{x+2y}{-3} = \frac{y+2z}{4} = \frac{2x+z}{5} = \lambda$ then $x = -3\lambda - 2y$, $y = 4\lambda - 2z$, $z = 5\lambda - 2x$

So $x = -3\lambda - 2(4\lambda - 2z)$, $x = -11\lambda + 4z$

$z = 5\lambda - 2(-11\lambda + 4z)$, $z = 3\lambda$ and $y = -2\lambda$

From $x + y + z = 2$, we have $\lambda - 2\lambda + 3\lambda = 2$ thus $\lambda = 1$

$x = 1,\ y = -2,\ z = 3$

Comment: Popular question, a MUST do to all.

11a) The sum of two numbers is 24. Find the two numbers if the sum of their squares is to be minimum.

Let the numbers be x and y.

$x + y = 24$, Let the sum be S such that $S = x^2 + y^2$

$S = x^2 + (24 - x)^2 = 2x^2 - 48x + 576$

For max or min, $\frac{dS}{dx} = 0$, i.e $4x - 48 = 0,\ x = 12$ Thus the numbers are $x = 12,\ y = 12$.

Comment: Analysis still on application of maximum and minimum. Poorly done yet so simple and straight forward.

b) A point P on the curve is given parametrically by $x = 3 - \cos\theta$ and $y = 2 + \sec\theta$.

Find the:

i) equation of the normal to the curve at the point $\theta = \frac{\pi}{3}$

When $\theta = \frac{\pi}{3}$, $x = 3 - \cos\frac{\pi}{3} = \frac{5}{2}$ and $y = 2 + \sec\frac{\pi}{3} = 4$, thus the point is $P\left(\frac{5}{2},\ 4\right)$.

$\frac{dx}{d\theta} = \sin\theta$ and $\frac{dy}{d\theta} = \sec\theta\tan\theta$ thus the gradient function is given by

$\frac{dy}{dx} = \sec\theta\tan\theta \times \frac{1}{\sin\theta} = \frac{1}{\cos^2\theta}$ when $\theta = \frac{\pi}{3}$, $\frac{dy}{dx} = 4$

Gradient of the normal is $-\frac{1}{4}$, so the equation of the normal is

$\frac{y - 4}{x - \frac{5}{2}} = -\frac{1}{4}$ To get $8y + 2x = 37$

ii) Cartesian equation of the curve.

$\cos\theta = 3 - x$, and from $y = 2 + \frac{1}{\cos\theta}$, so $\cos\theta = \frac{1}{y - 2}$

Thus $3 - x = \frac{1}{y - 2}$ to get $3y + 2x - xy = 7$

Comment: Popular question, a MUST do to all. Well done by most of the students.

12a) Solve the equation $2\tan\theta + \sin 2\theta\sec\theta = 1 + \sec\theta$ in the range $0^\circ \le \theta \le 2\pi$.

$$\frac{2\sin\theta}{\cos\theta}+\frac{2\sin\theta\cos\theta}{\cos\theta}=\frac{\cos\theta+1}{\cos\theta}$$

$(1+\cos\theta)(2\sin\theta-1)=0$ Either $\cos\theta=-1$, $\theta=\pi$ or $\sin\theta=\frac{1}{2}$, $\theta=\frac{\pi}{6}$, $\frac{5\pi}{6}$

Comment: **Most students cancelled the trigonometric function and missed the marks. Those who managed to get some angles did not convert them to radians.**

b) Prove that $\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$, hence or otherwise solve the equation $\tan(\theta-45^o)=6\tan\theta$, where $-180^o\leq\theta\leq180^o$.

From the LHS, $\tan(A-B)=\frac{\sin(A-B)}{\cos(A-B)}$

$=\frac{\sin A\cos B-\cos A\sin B}{\cos A\cos B+\sin A\sin B}$, divide through by $\cos A\cos B$

$=\frac{\tan A-\tan B}{1+\tan A\tan B}$ as R.H.S

For $A\equiv\theta,\ B\equiv45^o$, for $\tan(\theta-45^o)=6\tan\theta$, we get

$\frac{\tan\theta-\tan45}{1+\tan\theta\tan45}=6\tan\theta$, $\frac{\tan\theta-1}{1+\tan\theta}=6\tan\theta$

$6\tan^2\theta+5\tan\theta+1=0$, $6\tan^2\theta+3\tan\theta+2\tan\theta+1=0$

$(3\tan\theta+1)(2\tan\theta+1)=0$ $\tan\theta=-\frac{1}{3}$, $\theta=-18.43^o$, 161.57^o

$\tan\theta=-\frac{1}{2}$, $\theta=-26.57^o$, 153.43^o

Comment: **Popular question, a MUST do to all. Well done.**

13a) The vector equation of two lines are $\mathbf{r}=\begin{pmatrix}2\\1\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\2\end{pmatrix}$ and $\mathbf{r}=\begin{pmatrix}2\\2\\t\end{pmatrix}+\mu\begin{pmatrix}1\\2\\1\end{pmatrix}$ where t is a constant. If the two lines intersect find:

(i) t and the position vector of the point of intersection.

$\mathbf{r}=\begin{pmatrix}2+\lambda\\1+\lambda\\2\lambda\end{pmatrix}$ and $\mathbf{r}=\begin{pmatrix}2+\mu\\2+2\mu\\t+\mu\end{pmatrix}$, if they do intersect then $\mathbf{r}=\mathbf{r}$

Thus $\begin{pmatrix}2+\lambda\\1+\lambda\\2\lambda\end{pmatrix}=\begin{pmatrix}2+\mu\\2+2\mu\\t+\mu\end{pmatrix}$ implies that $2+\lambda=2+\mu$, **so** $\lambda=\mu$

Also $1+\lambda=2+2\mu$, for $\lambda=\mu$, then $1+\mu=2+2\mu$ so, $\mu=-1=\lambda$

Using $2\lambda = t + \mu$, then $-2 = t + -1$, gives $t = -1$

Position vector for point of intersection is given by $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$ or $\mathbf{r} = 2\mathbf{i} - 2\mathbf{k}$.

(ii) the angle between the two lines giving your answer to the nearest degree.

Let $\mathbf{d}_1 = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $\mathbf{d}_2 = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$ be the directional vectors.

Thus the angle is given by $\cos\theta = \dfrac{(\mathbf{i} + \mathbf{j} + 2\mathbf{k}).(\mathbf{i} + 2\mathbf{j} + \mathbf{k})}{\sqrt{1^2 + 1^2 + 2^2}\,.\sqrt{1^2 + 2^2 + 1^2}} = \dfrac{5}{6}$

$\cos\theta = \dfrac{5}{6}$, $\theta = 33.56^o \approx 34^o$ to the nearest degree.

b) The position vector of points P and Q are $2i - 3j + 4k$ and $3i - 7j + 12k$ respectively. Determine;

i) the size of PQ.

ii) The Cartesian equation of PQ.

i) PQ = $\begin{pmatrix} -2 \\ 3 \\ -4 \end{pmatrix} + \begin{pmatrix} 3 \\ -7 \\ 12 \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \\ 8 \end{pmatrix}$ $\therefore$ $|PQ|$ = $\sqrt{(1^2 + (-4)^2 + 8^2}$ = 9

ii) Vector equation PQ is $r_{PQ} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -4 \\ 8 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2+\lambda \\ -3-4\lambda \\ 4+8\lambda \end{pmatrix}$ $\therefore$ Cartesian equation is; $\dfrac{x-2}{1} = \dfrac{y+3}{-4} = \dfrac{z-4}{8} (= \lambda)$

Comment: Popular question, a MUST do to all. Well done question.

14a) A polynomial function $P(x)$ has a factor of $x^2 - 3x - 4$ and leaves a remainder of 5 when divided by $x + 2$. Determine the remainder when the polynomial is divided by $(x^2 - 3x - 4)(x + 2)$.

$P(x) \equiv (x-4)(x+1)(x+2)Q(x) + ax^2 + bx + c$

$P(-2) \equiv\ 4a - 2b + c = 5$ $\qquad P(-1) \equiv\ a - b + c = 0$

$P(4) \equiv\ 16a + 4b + c = 0$

Solve to get $a = \dfrac{5}{6}, b = \dfrac{-5}{2}, c = \dfrac{-10}{3}$, thus $R(x) = \dfrac{5}{6}x^2 - \dfrac{5}{2}x - \dfrac{10}{3}$

Comment: **Popular question, but the majority failed to interpret the question. Remainder theorem.**

b) Solve for x and y: $\begin{matrix} x\log_4 128 - y\log_8 2 = 6 \\ \log_2 x + \frac{1}{3}\log_2 y^3 = 2\log_4 6 \end{matrix}$

$\dfrac{x\log_2 128}{\log_2 4} - \dfrac{y\log_2 2}{\log_2 8} = 6$ we get; $\dfrac{7x}{2} - \dfrac{y}{3} = 6$ .....(i)

$\log_2 xy = \log_2 6$ $\Rightarrow$ $xy = 6$ ....(ii)

So $\dfrac{7}{2}.\dfrac{6}{y} - \dfrac{y}{3} = 6$ $\Leftrightarrow$ $y^2 + 18y - 63 = 0$

$(y+21)(y-3) = 0$ $y = 3, x = 2$ $y = -21, x = \frac{-2}{7}$

Comment: **Popular question but the students did not use or apply the rules of logarithms.**

15a) The curve $y = ax^2 + bx + c$ has a maximum point at $(2,\ 18)$ and passes through the point $(0,\ 10)$. Find the values of a, b, c.

$x = 0,\ y = 10$ so, $c = 10$

$\dfrac{dy}{dx} = 2ax + b$, $x = 2,\ 4a + b = 0$

$x = 2,\ y = 18$ so, $4a + 2b + c = 18$, $\Rightarrow 4a + 2b = 8$ $b = 8,\ a = -2$

Comment: **Poorly done yet it was so straight forward.**

b) Find the equations of the tangent and normal to the curve $x^2 + 3y^2 = 2a^2$ at the point $\left(a,\ \dfrac{a}{\sqrt{3}}\right)$.

$2x + 6y\dfrac{dy}{dx} = 0$, so, $\dfrac{dy}{dx} = -\dfrac{x}{3y}$, thus, at $\left(a,\ \dfrac{a}{\sqrt{3}}\right)$ grad of T is $-\dfrac{\sqrt{3}}{3}$

Equation is $\dfrac{y - \frac{a}{\sqrt{3}}}{x - a} = -\dfrac{\sqrt{3}}{3}$, $3y = -\sqrt{3}x + 2\sqrt{3}a$

Grad of normal is $\sqrt{3}$, thus equation of normal is

$\dfrac{y - \frac{a}{\sqrt{3}}}{x - a} = \sqrt{3}$, equation is $3y = 3\sqrt{3}x - 2\sqrt{3}a$.

Comment: **Popular question, but many failed to differentiate a constant to get zero.**